gravitational anomalies:

All species of fermions interact with gravitation in the same way.

Anomaly of the current $\bar{\chi} T \gamma^\mu \chi$ in the presence of gravitational field yields

$$\partial_\mu (\bar{\chi} T \gamma^\mu \chi) \sim tr\{T\} \varepsilon^{\mu\nu\rho\sigma} \underbrace{R_{\mu\nu\kappa\lambda}}_{\substack{\text{gravitational} \\ \text{field strength}}} R_{\rho\sigma}{}^{\kappa\lambda}$$

$\longrightarrow$ to avoid gravitational violation of gauge symmetry, need:

$$tr\{T_\alpha\} = 0 \qquad (1)$$

automatic for generators of "simple" algebras like $SU(2)$ or $SU(3)$

($tr\{T_\alpha\}$ is just a number $\rightarrow$ commutes with all T_β, so if non-zero then algebra is not simple)

$\longrightarrow$ need to check eq. (1) for $U(1)$ generators:

$$\sum Y/g' = 6\left(-\frac{1}{6}\right) + 3\left(\frac{2}{3}\right) + 3\left(-\frac{1}{3}\right) + 2\left(\frac{1}{2}\right) + (-1) = 0$$

$\longrightarrow$ no gravitational anomalies in Standard Model!

quite remarkable as Y/g'-values where deduced first from experiment!

<u>Question:</u> Is this just an accident or is there an underlying reason for the hypercharge assignment?

→ to answer assign arbitrary weak hypercharges a, b, c, d, and e to (u_L, d_L), u_R^*, d_R^*, (ν_L, e_L), and e_R^*, respectively.

Then anomaly cancellation tells us:

Fermions	SU(3)	SU(2)	U(1) $[Y/g']$
$\begin{pmatrix} u \\ d \end{pmatrix}_L$	3	2	a
u_R^*	$\bar{3}$	1	b
d_R^*	$\bar{3}$	1	c
$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$	1	2	d
e_R^*	1	1	e

- SU(3) – SU(3) – U(1):

$$\text{anomaly} \propto \sum_{3,\bar{3}} Y = 2a + b + c = 0$$

recall

$$D_{\alpha\beta\gamma} = \frac{1}{2} \mathrm{tr}\left[\{T_\alpha, T_\beta\} T_\gamma\right]$$
$$= \sum_a \frac{1}{2} \mathrm{tr}_{\chi_a}\left[\{T_\alpha, T_\beta\} T_\gamma\right]$$

where χ_a now runs over the fields in the table.

For example, for tr_{u_L} we get:

$$T_\alpha = \begin{array}{cc} \begin{pmatrix} t_\alpha & & & & & & \\ & t_\alpha & & & 0 & & \\ & & \bar{t}_\alpha & & & & \\ & & & \bar{t}_\alpha & & & \\ & 0 & & & 1 & & \\ & & & & & 1 & \\ & & & & & & 1 \end{pmatrix} & \begin{array}{l} u_L \\ d_L \\ u_R^* \\ d_R^* \\ (\check{e}) \\ \\ e_R^* \end{array} \end{array}$$

$$\longrightarrow \frac{1}{2} \mathrm{tr}\left[\{t_\alpha, t_\beta\} a \mathbb{1}_{3\times 3}\right]$$

where t_α is now $SU(3)$ generator $(\alpha = 1, \dots, 8)$

$$= a \underbrace{\mathrm{tr}[t_\alpha t_\beta]}_{= 3\delta_{\alpha\beta}}$$

similarly for the other fermions

- $SU(2) - SU(2) - U(1)$:

Here
$$T_\alpha = \begin{pmatrix} \sigma_\alpha & & & \\ & \sigma_\alpha & & 0 \\ & & \sigma_\alpha & \\ & 0 & & \ddots \end{pmatrix}$$
where σ_α is 2×2 Pauli matrix

$$\rightarrow \sum_{\text{doublets}} Y = 3a + d = 0$$

- $U(1) - U(1) - U(1)$:

$$T_\alpha = \begin{pmatrix} a\,\mathbb{1}_{6\times6} & & & & 0 \\ & b\,\mathbb{1}_{3\times3} & & & \\ & & c\,\mathbb{1}_{3\times3} & & \\ 0 & & & d\,\mathbb{1}_{2\times2} & \\ & & & & e \end{pmatrix}$$

$$\rightarrow \sum Y^3 = 6a^3 + 3b^3 + 3c^3 + 2d^3 + e^3 = 0$$

- graviton-graviton-$U(1)$:

$$\sum Y = 6a + 3b + 3c + 2d + e = 0$$

The above eqs. have only 2 solutions:

$U(1)$: $b/a = -4$, $\frac{c}{a} = 2$, $d/a = -3$, $e/a = 6$

$U(1)'$: $b = -c$, $a = c = d = e = 0$

Can not suppose that both $U(1)$ and $U(1)'$ are local symmetries: would encounter

$U(1)' - U(1)' - U(1)$ anomaly $\sim (-4) + (+2) \neq 0$

and $U(1)' - U(1) - U(1)$ anomaly $\sim (-4)^2 - (+2)^2 \neq 0$

The symmetry $U(1)$ is just the weak hypercharge of standard model
$U(1)'$ symmetry resembles nothing observed in nature.

Hidden sector:

There still maybe other $U(1)'$ gauge bosons that couple to other undetected $(SU(3)\times SU(2)\times U(1))$-neutral fermions as well as the known quarks and leptons
$\longrightarrow$ denote $U(1)'$ quantum numbers Y' of the multiplets (u_L, d_L), u_R^*, d_R^*, (ν_L, e_L), and e_R^* as a', b', c', d', and e'
no knowlegde of "hidden" fermions
$\rightarrow$ cancellation of $U(1)'-U(1)'-U(1)'$ and graviton-graviton-$U(1)'$ anomalies gives no constraints on a', b', c', d', or e'.
$\rightarrow$ remaining anomalies give:

- $SU(3)-SU(3)-U(1)'$:

$$\sum_{3,\bar{3}} Y' = 2a' + b' + c' = 0$$

- $SU(2)-SU(2)-U(1)'$: $\sum_{\text{doublets}} Y' = 3a' + d' = 0$

- $U(1)-U(1)-U(1)'$:

$$\sum Y^2 Y' = 6a' + 3(-4)^2 b' + 3(2)^2 c' + 2(-3)^2 d' + (6)^2 e' = 0$$

- $U(1)-U(1)'-U(1)'$:

$$\sum Y Y'^2 = 6a'^2 + 3(-4) b'^2 + 3(2) c'^2 + 2(-3) d'^2 + (6) e'^2 = 0$$

$\rightarrow$ general solution gives Y' as a linear combination of Y and $B-L$ quantum numbers
(B and L are conventional baryon and lepton numbers) with $B-L$ given by:

$1/3$	$-1/3$	$-1/3$	-1	$+1$
(u_L, d_L)	u_R^*	d_R^*	(ν_L, e_L)	e_R^*

$\rightsquigarrow$ if $B-L$ where a local symmetry, it has to be broken in the IR as ordinary bodies have macroscopic values of $B-L$.

$\rightsquigarrow$ possible neutral vector boson somewhat heavier than Z^0

§ 6.4 Massless Bound States

– 't Hooft anomaly matching

QCD has massless or very light composite particles in IR (Pions)

<u>Question</u>: What is the criterion for this to happen?

Answer was provided by 't Hooft:

If underlying (UV) theory has global chiral symmetries consisting of trfs. on elementary left-handed spin $\frac{1}{2}$ fermions χ with symmetry generators T_α, T_β, etc. and if

$$tr\left[\{T_\alpha, T_\beta\} T_\gamma\right] \neq 0$$

then in the IR the spectrum of bound states <u>must</u> include spin $\frac{1}{2}$ massless particles transforming under the same symmetries with generators $\mathcal{T}_\alpha$, $\mathcal{T}_\beta$, etc., s.t.

$$\mathrm{tr}\left[\{\tilde{T}_\alpha, \tilde{T}_\beta\}\tilde{T}_\gamma\right] = \mathrm{tr}\left[\{T_\alpha, T_\beta\}T_\gamma\right]$$

or in other words:

"the massless spin $\frac{1}{2}$ bound states reproduce the anomalies of the trapped elementary spin $\frac{1}{2}$ fermions of which they are composed."